Leiden Math trail

The Morspoort in Leiden

Colophon

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This math trail has been put together with the greatest care. However, the authors can in no way be held responsible or liable for matters concerning the content of the mathematical walking tour.

References (Accessed 15-01-2023)
Lottery: http://www.fransmensonides.nl/janvanhout1.htm
Leiden fortress: http://www.leiden-toerisme.nl/zien-doen-beleven/bezienswaardigheden/burcht/
Snellius: https://nl.wikipedia.org/wiki/Ibn_Sahl_(wiskundige)
Gunpowder disaster: http://www.leidenuniv.nl/nieuwsarchief2/1370.html
Pieterskerk: http://www.pieterskerk.com/nl/
Cannon: http://wm.math4allview.appspot.com/MathUnited/view?comp=lj3-v-h29&subcomp=lj3-v-h29-09&variant=basis_wm

Introduction

Hi there!

Welcome to the Leiden math trail. This walk will take you along a number of beautiful, special places in the city, places that will reveal some of the mathematics hidden below the city's surface. There are all sorts of different questions and puzzles. We hope you enjoy it!

Here is what you need:

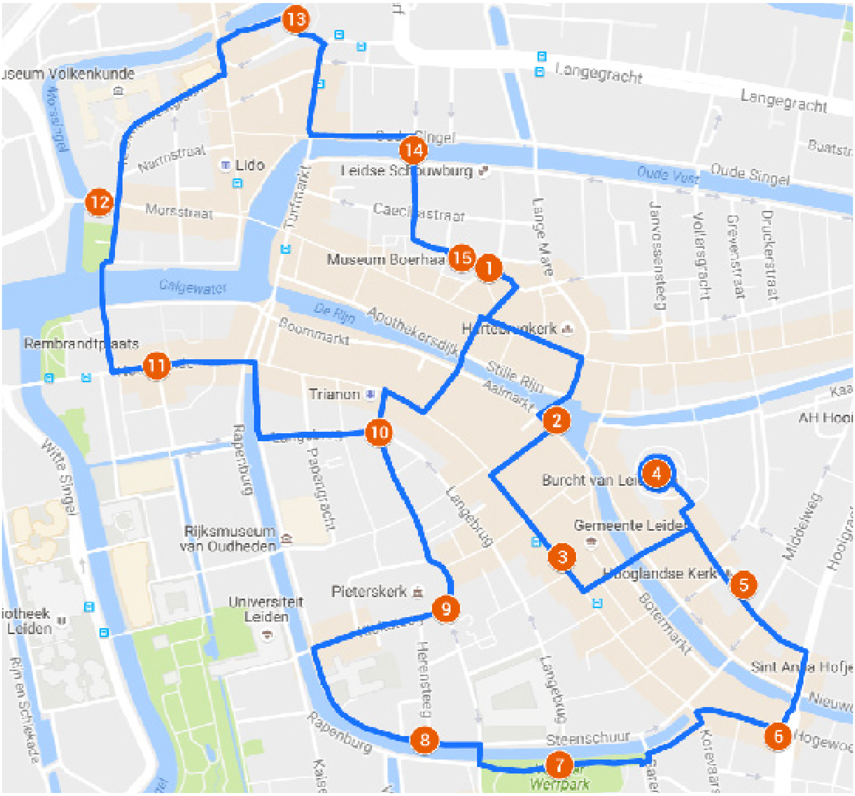
- a calculator (or a phone with a calculator app)
- a ruler or set square (if you have not got one with you, there is a printed version at the end of the booklet)
- a pen or a pencil to write down the answers

The route begins and ends at Rijksmuseum Boerhaave (Lange Sint Agnietenstraat 10). The standard route (questions 1–10) takes 1½ to 2 hours, and the extended route (questions 1–14) takes an additional 45 minutes. At the end of both routes is a bonus question, a fun puzzle about the bridges of Leiden. At the end of the booklet you will find several blank pages for your notes and calculations.

Have fun!

Route

The route begins and ends at the Rijksmuseum Boerhaave (Lange Sint Agnietenstraat 10). The normal route (questions 1–10) takes 1½ to 2 hours, and the extended route (questions 1–14) takes an additional 45 minutes. Question 15 is the bonus question.



Start of the math trail

Do question 1 at the entrance of Rijksmuseum Boerhaave.

Question 1
The lottery

In 1596 gambling was illegal, but lotteries for charity were allowed. Churches in Leiden encouraged people to buy lottery tickets, and the people of Leiden bought tons of them. The churches sold a total of 281,232 lottery tickets. Tickets cost 30 cents apiece, and there were 731 prizes. The jackpot was 1,500 guilders, which at the time was six times what an average labourer earned in one year. After the cash prizes were distributed, 52,455 guilders remained. That money was used to rebuild the St Caecilia Convent, which has been the home of Rijksmuseum Boerhaave since 1991.

A. What are your chances of winning a prize if you have bought a single lottery ticket?

B. The chances of winning the jackpot are 1 in 281,232. How many dice do you need so that your chances of rolling all sixes is the same as your chances of winning the jackpot?

a. 7 dice
b. 14 dice
c. 20 dice

C. What percentage of the money made from ticket sales was used for the prize money?

Route to question 2
Standing with your back to Rijksmuseum Boerhaave, turn left. Walk across the square (Vrouwenkerkhof), keeping to your left, continue walking into the street called Vrouwenkerkkoorstraat. At the end, make a right into the street called Lange Mare. Pass by the church on your right, cross the Haarlemmerstraat, and walk up to the bridge (Catharinabrug). Walk onto the bridge and while you are there, do question 2.

Question 2.
The Weighhouse

On the Catharinabrug, if you look up and to your left, you will see the Weighhouse (de Waag). This was the place where goods, brought in by merchants from other cities, were weighed and inspected. A merchant was obliged to pay a weighing fee each time his goods were weighed. These goods were weighed using a set of scales with two weighing pans, see illustration. You can also see a balance depicted in the relief on the building. With a weighing scale you can only tell whether the goods on one side are heavier than the other side, or whether they are equal in weight. This type of scale does not produce a numerical value for the weight of the goods.

Suppose you have nine balls, all of the same colour, shape and size, but one ball is heavier than all the others. How many times do you need to weigh to figure out which of the balls is heavier? Try to figure out a way to pay as little in weighing fees as possible.

Route to question 3
Go over the bridge to your left and then immediately to the right at the Weighing House into the alley called Mandenmakerssteeg. The alley leads to the Breestraat. Go to the left at this street and walk towards the city hall. Before you get to the city hall steps, you will see an iron bar mounted to the left of the wooden door.

Question 3.

The Rijnland foot

The iron bar is a ‘foot’, an old-fashioned unit of length. Willebrord Snel van Royen (Snellius), a Leiden mathematician, determined the earth’s circumference using this unit of measurement.

A. The earth’s circumference is 40,000 km. How many feet is that?

B. If you take a single step, how many feet have you moved forward?

C. How many steps do you need to take to go once around the earth?

More about... the foot

The ‘foot’ mounted here on city hall is a unit of length that was used in the past. Twelve feet made a ‘rod’. If you look closely, you will see some horizontal lines on the wall under the iron bar. Those lines represent twelve feet, and together they amount to one rod. The Netherlands officially started using these units of measurement in 1808. Eight years later, the Netherlands introduced the metre as still used today.

Route to question 4
Walk further along city hall and turn left at Koornbrugsteeg. Go straight over the bridge and into the Burgsteeg. At the end of this alley, turn to the left and go through the gate. Walk between the pillars and up the stairs to the fortress.

Question 4.

The fortress

The primary purpose of Leiden's fortress, or burcht, was to protect city residents in times of war. It was built in the 11th century and is one of the oldest fortresses in the Netherlands.

A. Estimate how high the walls are and what the fortress's diameter is. What is the volume of the fortress?

B. A cannonball has a diameter of 15 centimetres. You can calculate the volume of a ball using the following formula: $^{4}/_{3}\, \pi\, r^3$. Suppose you want to use the fortress for storing cannonballs up to right under the crenels (the openings at the top of the walls). How many cannonballs could you fit in there?

a. 26000 cannonballs
b. 260000 cannonballs
c. 2600000 cannonballs
d. 26000000 cannonballs

More about... the fortress

Countess Ada van Holland fought an important battle against her uncle at the fortress. Ada's father, Count Dirk VII, died in 1203 and left his position to his daughter Ada. Her uncle Willem I van Holland did not approve of this and wanted to become count himself. Ada sought protection in the fortress. A fierce battle ensued between Ada's army and that of her uncle. Ada eventually surrendered due to a lack of food.

Route to question 5
Walk back down and go right into the street you took to get here. Go back through the gate and straight into the Nieuwstraat. Do question 5 standing on the little square to your left, in front of the Hooglandse church.

Question 5.

Leiden's coat of arms

In Leiden's coat of arms, which you can spot in the triangle above the door, three colours are used: white, red and gold. The creators of the coat of arms could have alternatively chosen to use golden keys, a white crown and a red background.

A. How many different combinations can you make if you only have these three colours and you want the keys, the background and the crown to each be of a different colour?

B. How many combinations are there if you have four colours and paint the two keys a different colour?

C. Think about how that would extend to other numbers of colours and objects. For instance, how many possible combinations would there be if you had 80 different colours and 80 different objects?

More about... Leiden's coat of arms

'The city of keys' is one of Leiden's nicknames. This nickname can be traced back to St Peter, the city's patron saint. Peter always had two keys with him, the keys to heaven. That is why Leiden's coat of arms and its flag feature a pair of keys.

Route to question 6
Walk across the Nieuwstraat and when you get to the end of the street go to your right, walking up the Hooigracht. Then cross the bridge into the Watersteeg. Cross over the bike path. Be careful; it is often quite busy here. To the right on the corner you will see a formula on the wall.

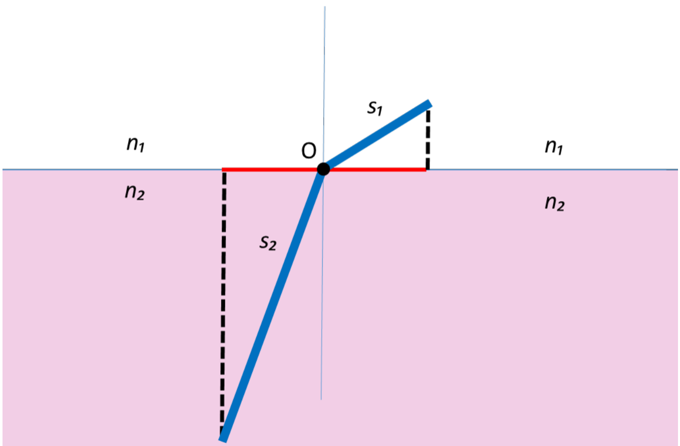
Question 6.

Snell's law

The formula on the wall is Snell's law. One can use this formula to determine the refraction of light from one material to another. The white area represents air. We will now follow a step-by-step procedure to calculate the material in the pink area.

Step-by-step procedure

1. Take two segments of equal length on the horizontal axis from point O (the red lines) and draw a vertical dotted line from s_1 and s_2 as indicated in the drawing. Measure the length of the beam of light of the air and of the other material (s_1 and s_2) in the drawing.
2. The relation between the so-called refraction indices (indicated with n, see table) of the two materials is equal to the relation between s_1 and s_2. So, $n_1/n_2 = s_1/s_2$. Calculate n_2 by measuring s_1 and s_2 and place them in the formula.
3. What material is the pink area? Use the table on the following page.



Material	n
Air	1.00
Water	1.33
Glass	1.50
Diamond	2.42

More about... Snell's law

Snell's law is named after the Leiden mathematician Snellius, although the Arab mathematician Ibn Sahl probably discovered the law earlier. It is a physical formula relating to the refraction of light. If you look at your own hand under water, it appears as if your hand is in a different place than where it should be. That is because light slightly changes direction when it passes from one material (water) to another (air).

Route to question 7

Go into the street with the busy cycle path, called Hogewoerd. At the end of Hogewoerd, cross the street, turn slightly to your left into Steenschuur. The canal should be on your right-hand side. Go straight into the Van der Werf Park. When you have walked past the sculpture, on the other side of the canal is a commemorative plaque on the quayside. The next question is about the gunpowder disaster commemorated on the plaque.

Question 7.
The gunpowder disaster

At this spot, on 12 January 1807, a ship carrying 17,760 kg of gunpowder exploded. King Louis Bonaparte heard the blast all the way in The Hague (Den Haag). Sound travels at a speed of 340 metres per second. How long did it take for the blast to be heard in The Hague after it took place?

Need a break?
You can take a break in the Van der Werfpark, at the end is the Kamerlingh Parktuin & Cafe located. More restaurants and shops can be found at the end of the park left (Doezastraat).

Route to question 8
Walk out of the park and go to your right and over the bridge. Go immediately to the left, with the canal on your left. Walk down Rapenburg one block, up to the orange house (see photo).

Question 8.
The orange house

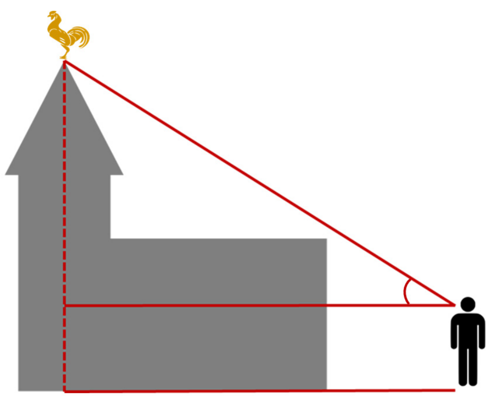
Suppose that we want to paint this orange house. We want to paint both walls facing the street. It takes one litre of paint to cover 10 m^2. How many litres do we need to paint this surface a different colour?

Route to question 9
Continue walking down Rapenburg, after the bend take a right turn into the alley called Kloksteeg. Keep walking, passing the church called Pieterskerk until you get to the corner with house numbers 38 and 40. Do question 9 here.

Question 9.
The Pieterskerk

The Pieterskerk has been Leiden's main church since time immemorial.

A. Try to estimate how tall the Pieterskerk is (up to the very top of the little tower with the golden rooster). (You may want to use the drawing with the red lines and SOSCASTOA).



B. The Pieterskerk once had a high tower (see picture). You can use a formula to calculate how far you would have been able to see from the tower. That formula is **distance = 3.6 * √height**, with the height in metres and the distance in kilometres. How far into the distance would you have been able to see from the large old tower, which was 110 metres tall? And how far from the small remaining tower?

More about... the Pieterskerk
The large old tower was 110 metres high so that sailors could see it from the sea. The tower collapsed in 1512. No other tower was built to replace it, but church bells were installed on the square in front of the church. The gunpowder disaster of 1807 broke all of the stained-glass windows; the current windows are 'modern'.

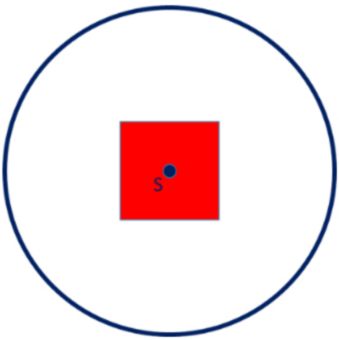
Route to question 10
Go around the church and walk straight into the street called Pieterskerkstraat. As you walk further, this street will turn into the Pieterskerkgracht. At the end of the Pieterskerkgracht, on the left corner, you will see a lamppost at house number 3. Do question 10 here.

Question 10.
The lantern

Look at the lamppost at the crossing, on the left side of the street. The little red square represents the top view of the lantern. Walk in a circle around the lamppost. Write a letter A on the edge of the circle when you see the lantern from that location as in photo A. Do the same for letter B and photo B. How big is the angle compared to S if you walk from A to B?

A.

B.



Continue with the extended walking tour

Route to question 11

After question 10, take a left and go into the street called Langebrug. Take a right when you get to the water. Take the first left, crossing the water, and walk down Noordeinde street until you get to the Oude Varkenmarkt. On the other side of the street you will see a red step-gabled house (Noordeinde nr. 24). Do question 11 here.

De Route back to Rijksmuseum Boerhaave

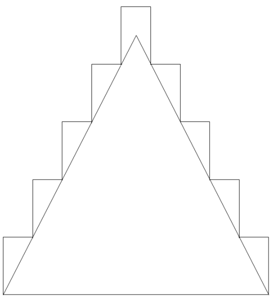
Go straight into the alley called Mooi Japiksteeg. Then to your right, into the Breestraat. Take the first left, going into the alley called Vrouwensteeg. Cross the bridge and walk straight down the Vrouwensteeg. Cross Haarlemmerstraat diagonally and walk into the Vrouwenkerkkoorstraat. On the little square, go to your left. Now you are back at the entrance to Rijksmuseum Boerhaave. While you are here, do the bonus question on page 22!

Question 11.
The stepped gable

A stepped gable is a traditional old housetop gable that gets narrower in a stepwise fashion as it goes higher.

A. A single brick costs 20 cents. How much more would you need to pay for a stepped gable as compared to a straight triangular gable?

B. Suppose that we make the steps of the gable just half as large (that is, half as wide and half as tall). Draw the new steps on the picture. Now how much more would you need to pay extra for this stepped gable as compared to a straight triangular gable?



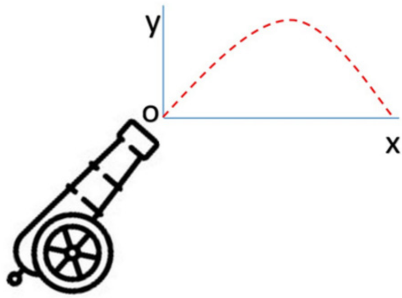
Route to question 12
Keep walking down the street called Noordeinde until you get to the pedestrian crossing. Cross and go straight into Weddesteeg. Keep walking and cross the bridge, keeping to your left, passing by the windmill and entering the little park. Go left, passing the building. To the right, in front of the gate, you will see a cannon.

Question 12.

The cannon

A cannon shoots a cannonball. The point where the cannonball leaves the shaft we can call O. We put the X-axis horizontally through O, and the Y-axis is vertical. The cannonball's path is (approximately) a parabola: $\mathbf{y = a * x - 0.01x^2}$, for a certain number a; x and y are in metres.

A. Determine the value of 'a' if the cannonball reaches 100 metres. (x = 100 metres)



B. What is the cannonball's maximum height if it reaches 100 metres? (x = 100 metres)

More about... the Morspoort
The Morspoort is one of Leiden's old city gates. At night the gates closed to keep the city safe. The Morspoort also served as a prison for a long period of time.

Route to question 13
Go through the gate (or to the left around the gate if you can not go through the gate) and turn immediately left onto Binnenvestgracht. Walk along with the bend to the right. Cross the street at the end. Take care; this is a busy street. Take a left and walk past the Van der Werff restaurant on your right. Before your reach the water, take a right onto the Kiekpad pedestrian path and enter the little park. Ahead of you, you will see De Valk windmill.

Question 13.
The windmill

They used to tie people to the windmill's vanes (its 'arms') as a punishment. Eventually, it became a fairground attraction.

A. Where would you prefer to be tied on the mill's vanes, at the middle or at the end? Where would you move more quickly?

B. What is the difference in speed (in km/hour) between the middle and the end of the vane if the vanes spin at a rate of 12 rotations per minute? Estimate what the vane's length is.

Route to question 14
Follow the Kiekpad towards the mill and cross the cyclepath. Keep right and cross the pedestrian crossing. Walk down the Nieuwe Beestenmarkt and until you reach the canal. Take a left before the canal, onto the Oude Singel. Walk onto the first little bridge and do question 14 while you are there.

Question 14.
Gables

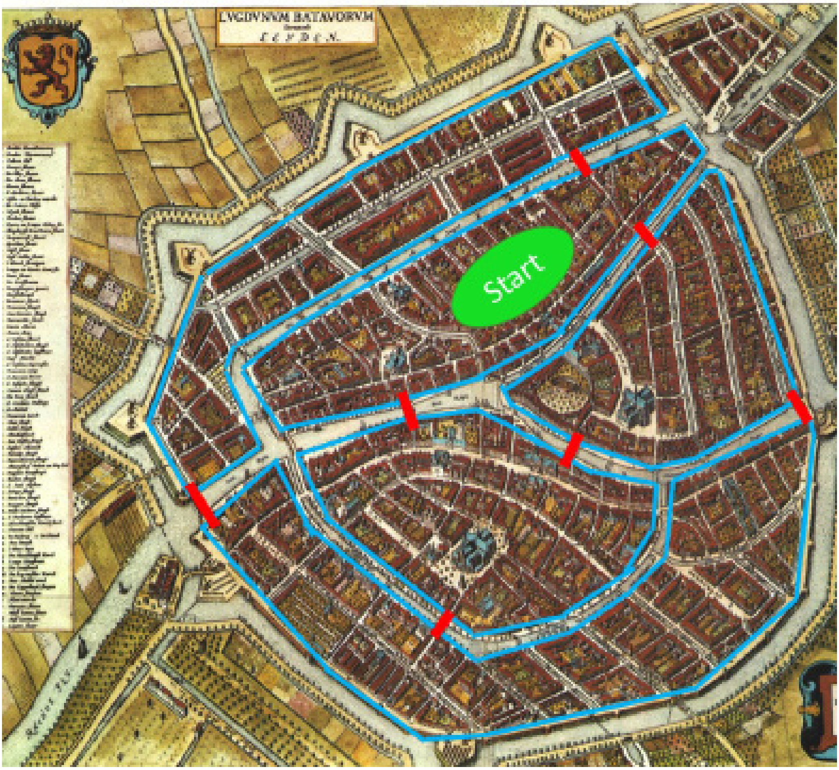
Stand on the middle of the bridge over the Oude Vest. On both sides of the bridge you will see another bridge. How many types of gables can you see between these two bridges, on both sides of the canal? And what different types of gables are there?
Make a table with the various types of gables and how many houses you see with each type.

Route back to Rijksmuseum Boerhaave
Walk over the bridge and straight into the alley called Lange Lijsberthsteeg. Go to the end of the street as it turns left, then following it right and immediately left. Now you are back at the entrance to Rijksmuseum Boerhaave. While you're here, do the bonus question on the following page!

Bonus question

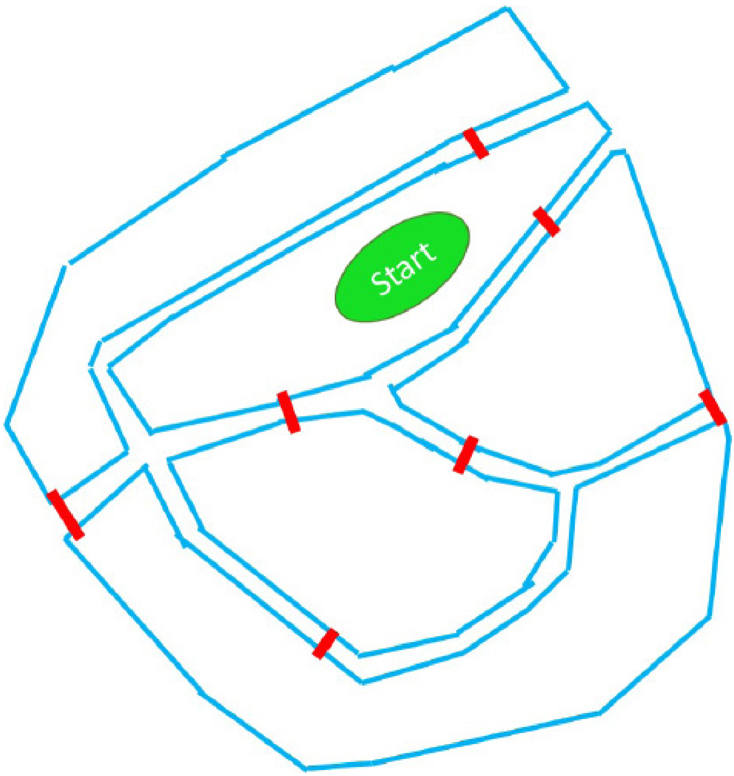
The bridge problem

Is it possible to take a walk through the city, passing through all five of Leiden's districts and crossing each red bridge exactly once? The starting point for your walk is the green dot. You can end in any district.
If it is possible, show the route. If it is not possible, where should you put a bridge to make it possible?

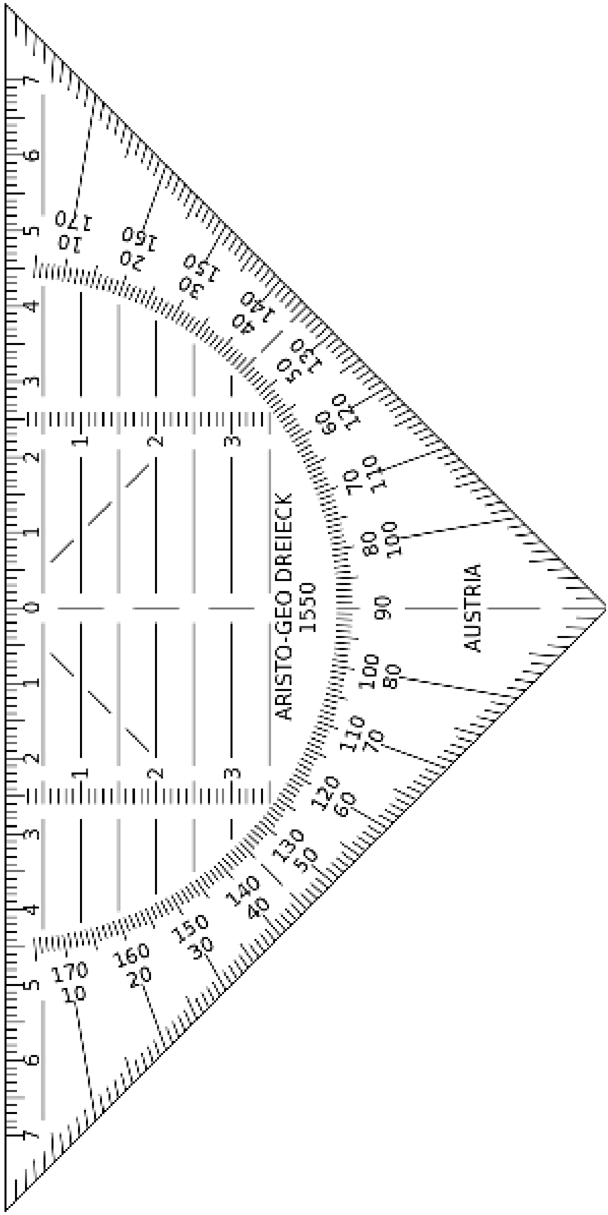


More about... the canals

In the 17th century the city was growing quite rapidly. Within 50 years the number of inhabitants had risen from 15,000 to 45,000. This was the result of large wave of immigrants coming from Flanders. These immigrants made a boom in the textile industry. This growth made it necessary to dig new canals. The canals as we see them today were constructed in 1659. At the time, Leiden was the largest city in the Netherlands after Amsterdam.



Set square



Scratch Paper

Scratch Paper

Scratch Paper

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$3{,}6 * \sqrt{h} \quad y = a * x - 0{,}01x^2 \quad a^2 + b^2 = c^2 \quad {}^4/_3 \pi r^3 \quad n_1/n_2 = s_1/s_2 \quad L = 3{,}6 * \sqrt{h} \quad y = a * x - 0{,}01$
$+ b^2 = c^2 \quad {}^4/_3 \pi r^3 \quad n_1/n_2 = s_1/s_2 \quad L = 3{,}6 * \sqrt{h} \quad y = a * x - 0{,}01x^2 \quad a^2 + b^2 = c^2 \quad {}^4/_3 \pi r^3 \quad n_1/n_2 =$
$/s_2 \quad L = 3{,}6 * \sqrt{h} \quad y = a * x - 0{,}01x^2 \quad a^2 + b^2 = c^2 \quad {}^4/_3 \pi r^3 \quad n_1/n_2 = s_1/s_2 \quad L = 3{,}6 * \sqrt{h} \quad y = a *$
$- 0{,}01x^2 \quad a^2 + b^2 = c^2 \quad {}^4/_3 \pi r^3 \quad n_1/n_2 = s_1/s_2 \quad L = 3{,}6 * \sqrt{h} \quad y = a * x - 0{,}01x^2 \quad a^2 + b^2 = c^2 \quad {}^4/_3$
$n_1/n_2 = s_1/s_2 \quad L = 3{,}6 * \sqrt{h} \quad y = a * x - 0{,}01x^2 \quad a^2 + b^2 = c^2 \quad {}^4/_3 \pi r^3 \quad n_1/n_2 = s_1/s_2 \quad L = 3{,}6 *$
$= a * x - 0{,}01x^2 \quad a^2 + b^2 = c^2 \quad {}^4/_3 \pi r^3 \quad n_1/n_2 = s_1/s_2 \quad L = 3{,}6 * \sqrt{h} \quad y = a * x - 0{,}01x^2 \quad a^2 + b^2 =$
$/_3 \pi r^3 \quad n_1/n_2 = s_1/s_2 \quad L = 3{,}6 * \sqrt{h} \quad y = a * x - 0{,}01x^2 \quad a^2 + b^2 = c^2 \quad {}^4/_3 \pi r^3 \quad n_1/n_2 = s_1/s_2 \quad L =$
$\sqrt{h} \quad y = a * x - 0{,}01x^2 \quad a^2 + b^2 = c^2 \quad {}^4/_3 \pi r^3 \quad n_1/n_2 = s_1/s_2 \quad L = 3{,}6 * \sqrt{h} \quad y = a * x - 0{,}01x^2 \quad a^2 +$
$c^2 \quad {}^4/_3 \pi r^3 \quad n_1/n_2 = s_1/s_2 \quad L = 3{,}6 * \sqrt{h} \quad y = a * x - 0{,}01x^2 \quad a^2 + b^2 = c^2 \quad {}^4/_3 \pi r^3 \quad n_1/n_2 = s_1/s_2$
$3{,}6 * \sqrt{h} \quad y = a * x - 0{,}01x^2 \quad a^2 + b^2 = c^2 \quad {}^4/_3 \pi r^3 \quad n_1/n_2 = s_1/s_2 \quad L = 3{,}6 * \sqrt{h} \quad y = a * x - 0{,}01$
$+ b^2 = c^2 \quad {}^4/_3 \pi r^3 \quad n_1/n_2 = s_1/s_2 \quad L = 3{,}6 * \sqrt{h} \quad y = a * x - 0{,}01x^2 \quad a^2 + b^2 = c^2 \quad {}^4/_3 \pi r^3 \quad n_1/n_2 =$
$/s_2 \quad L = 3{,}6 * \sqrt{h} \quad y = a * x - 0{,}01x^2 \quad a^2 + b^2 = c^2 \quad {}^4/_3 \pi r^3 \quad n_1/n_2 = s_1/s_2 \quad L = 3{,}6 * \sqrt{h} \quad y = a *$
$- 0{,}01x^2 \quad a^2 + b^2 = c^2 \quad {}^4/_3 \pi r^3 \quad n_1/n_2 = s_1/s_2 \quad L = 3{,}6 * \sqrt{h} \quad y = a * x - 0{,}01x^2 \quad a^2 + b^2 = c^2 \quad {}^4/_3$
$n_1/n_2 = s_1/s_2 \quad L = 3{,}6 * \sqrt{h} \quad y = a * x - 0{,}01x^2 \quad a^2 + b^2 = c^2 \quad {}^4/_3 \pi r^3 \quad n_1/n_2 = s_1/s_2 \quad L = 3{,}6 *$
$= a * x - 0{,}01x^2 \quad a^2 + b^2 = c^2 \quad {}^4/_3 \pi r^3 \quad n_1/n_2 = s_1/s_2 \quad L = 3{,}6 * \sqrt{h} \quad y = a * x - 0{,}01x^2 \quad a^2 + b^2 =$
$/_3 \pi r^3 \quad n_1/n_2 = s_1/s_2 \quad L = 3{,}6 * \sqrt{h} \quad y = a * x - 0{,}01x^2 \quad a^2 + b^2 = c^2 \quad {}^4/_3 \pi r^3 \quad n_1/n_2 = s_1/s_2 \quad L =$
$\sqrt{h} \quad y = a * x - 0{,}01x^2 \quad a^2 + b^2 = c^2 \quad {}^4/_3 \pi r^3 \quad n_1/n_2 = s_1/s_2 \quad L = 3{,}6 * \sqrt{h} \quad y = a * x - 0{,}01x^2 \quad a^2 +$
$c^2 \quad {}^4/_3 \pi r^3 \quad n_1/n_2 = s_1/s_2 \quad L = 3{,}6 * \sqrt{h} \quad y = a * x - 0{,}01x^2 \quad a^2 + b^2 = c^2 \quad {}^4/_3 \pi r^3 \quad n_1/n_2 = s_1/s_2$
$3{,}6 * \sqrt{h} \quad y = a * x - 0{,}01x^2 \quad a^2 + b^2 = c^2 \quad {}^4/_3 \pi r^3 \quad n_1/n_2 = s_1/s_2 \quad L = 3{,}6 * \sqrt{h} \quad y = a * x - 0{,}01$
$+ b^2 = c^2 \quad {}^4/_3 \pi r^3 \quad n_1/n_2 = s_1/s_2 \quad L = 3{,}6 * \sqrt{h} \quad y = a * x - 0{,}01x^2 \quad a^2 + b^2 = c^2 \quad {}^4/_3 \pi r^3 \quad n_1/n_2 =$